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**The Fractal Market Hypothesis:
Applications to Financial
Forecasting**

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Preface

Most financial modelling systems rely on an underlying hypothesis known as the Efficient Market Hypothesis (EMH) including the famous Black-Scholes formula for pricing an option. However, the EMH has a fundamental flaw; it is based on the assumption that economic processes are normally distributed and it has long been known that this is not the case. This fundamental assumption leads to a number of shortcomings associated with using the EMH to analyse financial data which includes failure to predict the future volatility of a market share value.

This book introduces a new financial risk assessment model based on Lévy statistics and considers a financial forecasting system that uses a solution to a non-stationary fractional diffusion equation characterized by the Lévy index. Variations in the Lévy index are considered in order to assess the future volatility of financial data together with the likelihood of the markets becoming bear or bull dominant, thereby providing a solution to securing an investment portfolio. The key hypothesis associated with this approach is that a change in the Lévy index precedes a change in the financial signal from which the index is computed and can therefore be used as a risk management index. It is shown that there is a quantitative relationship between Lévy's characteristic function and a random scaling fractal signal obtained through a Green's function solution to the fractional diffusion equation. In this sense, the model considered is based on the Fractal Market Hypothesis and two case studies are considered in this respect: (i) application of the hypothesis by predicting the volatility associated with foreign exchange markets; (ii) application to the ABX index.

